Logical Neural Networks

An Introduction - Dip our toes in LNN

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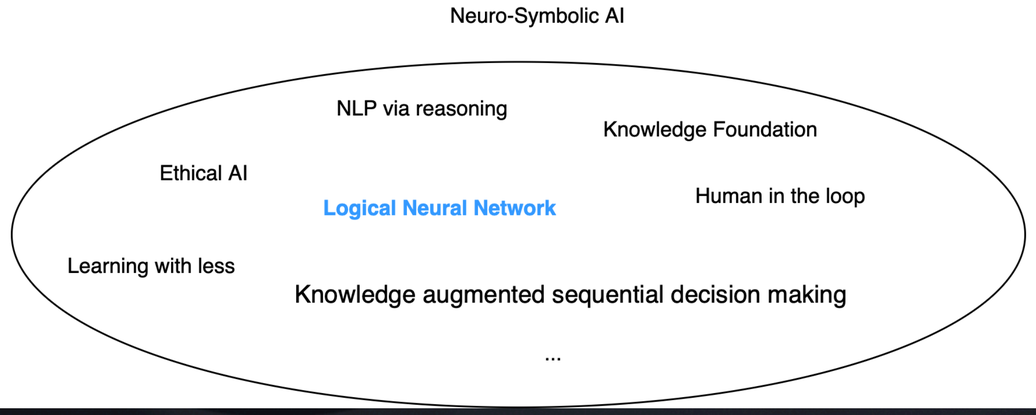
Neural Symbolic AI

An overview-NS

A higher level of the purpose

- *"In particular it is aimed at **augmenting (and retaining)** the strengths of statistical AI (machine learning) with the complementary capabilities of symbolic AI (knowledge and reasoning)."*
- *"...**revolution** rather than **evolution**."*



Ref: https://ibm.github.io/neuro-symbolic-ai/

An overview-NS

A higher level of the purpose

- *Solve harder problems.*
- *Learn with less data while maintain the ability to generalise to a large number of tasks.*
- *Provide white box reasoning on decision/action.*

Ref: https://ibm.github.io/neuro-symbolic-ai/

An overview-NS

News?

- *LLM Reasoning -Denny Zhou*
- *Teaching LLMs to Plan: Logical Chain-Of-Thought Instruction Tuning for Symbolic Planning*
  MIT CSAIL
- ...

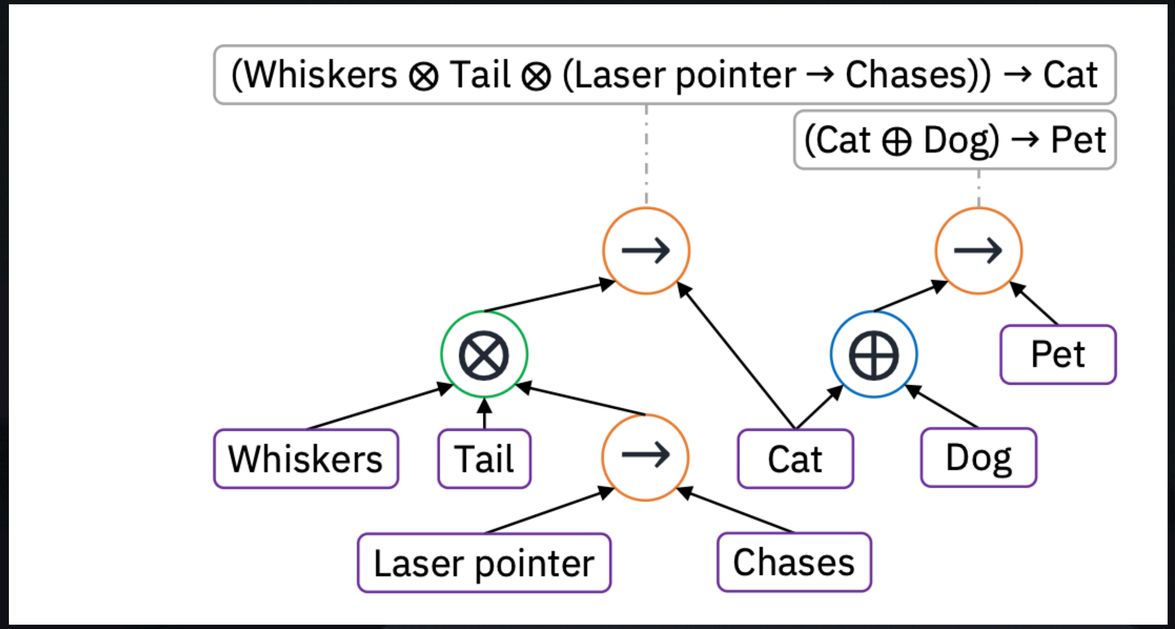
Ref: https://ibm.github.io/neuro-symbolic-ai/

Logical Neural Networks

What

What's the definition?

- Syntax trees
- Formulae
- Neurons for logical operation/proposition



⊗: Real value conjunction

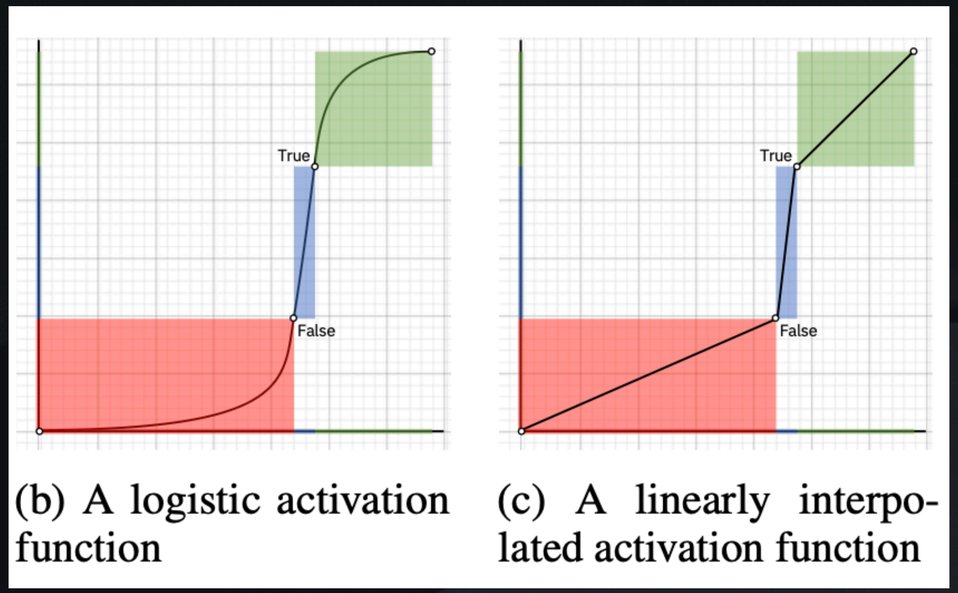
⊕: Real value disjunction

Ref: https://arxiv.org/pdf/2006.13155

What

What's the main difference? - Activation function

- Compare with classical activation function



(b) A logistic activation function

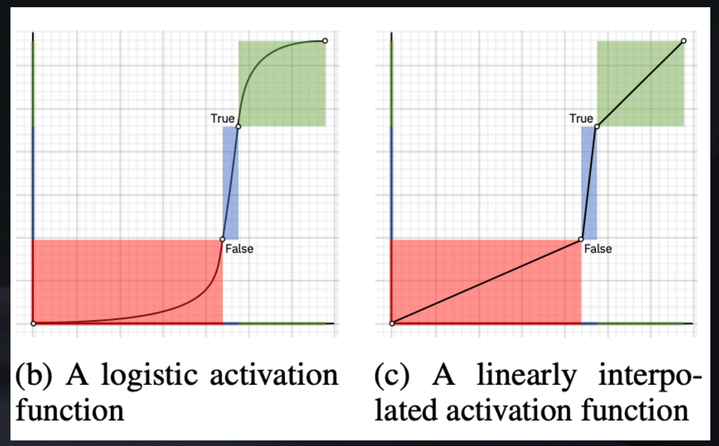
(c) A linearly interpolated activation function

Ref: https://arxiv.org/pdf/2006.13155

What

What's the main difference? - Activation function

- $\wedge$, $\vee$, $\neg$, $\rightarrow$ are implemented by constrained neural activation function
- Behaviours should be the same in classical exact inputs



(b) A logistic activation function

(c) A linearly interpolated activation function

$t-norm$ logic

$$T_G(a,b) = min(a,b) = b, if \quad b < a$$

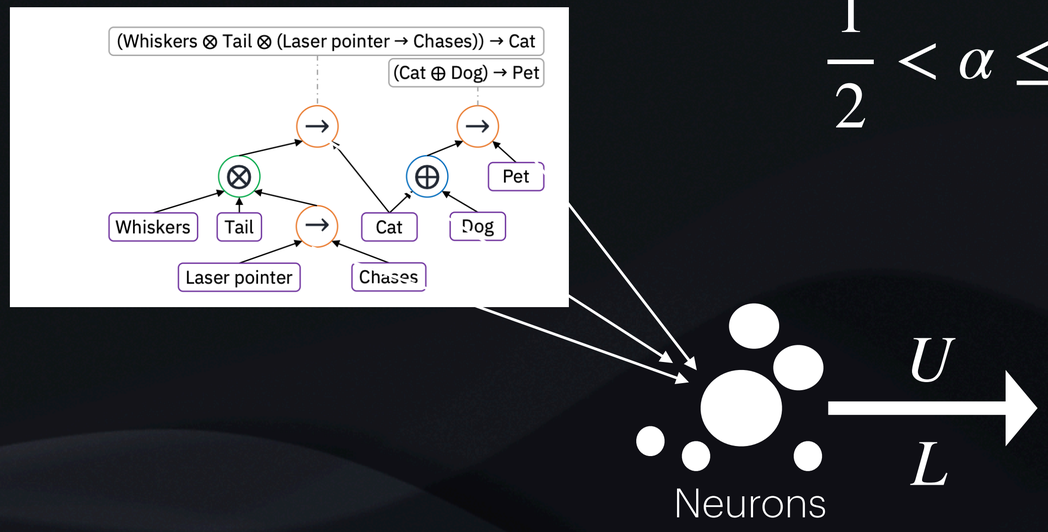
$$T_P(a,b) = a \times b$$

$$T_L(a,b) = max(0, a+b-1)$$

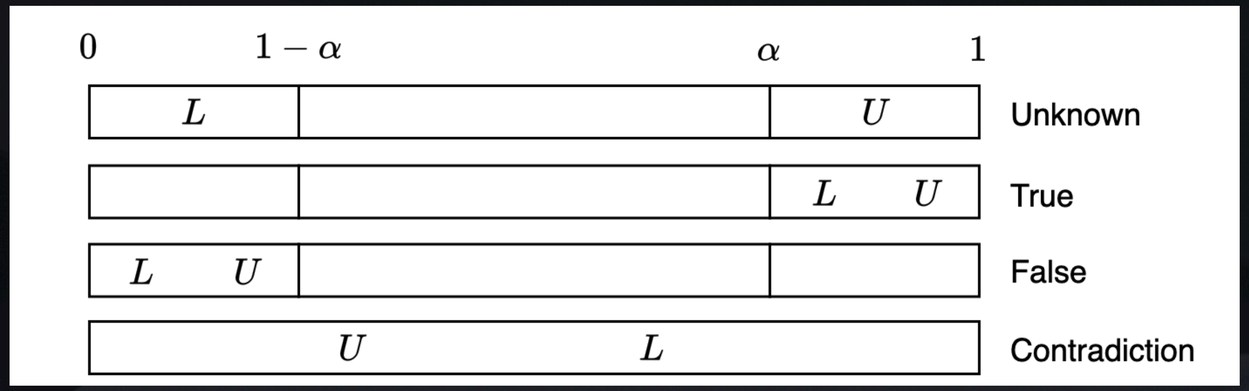
What

What's the main difference? - Bounds on results

- Truth values can be known/approximately known/unknown/contradictory states.



$\frac{1}{2} < \alpha \leq 1$ is a threshold value aided in evaluating the "truth".



What

What's the main difference? - Bidirectional inference

- Upwards: Normal propagate
- Downwards:

$x, x \to y \vdash \quad y$	(*modus ponens*)	$x, \neg(x \wedge y) \vdash \neg y$	(conjunctive syllogism)
$\neg y, x \to y \vdash \neg x$	(*modus tollens*)	$\neg x, \quad x \vee y \vdash \quad y$	(disjunctive syllogism)

Ref: https://arxiv.org/pdf/2006.13155

What

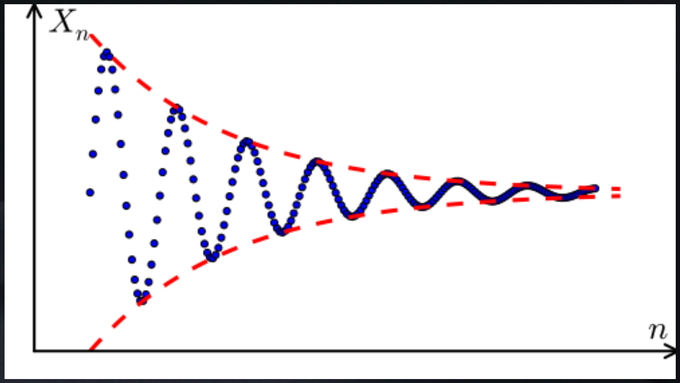
What's the main difference? - Bidirectional inference

- The Upward-Downward algorithm.
- Propagates truth value upwards and downwards, **tightening** the truth bounds.
  - Q: What if it drops into an infinite loop? Input becomes more true, output becomes less true? Will this happen?

What

What's the main difference? - Bidirectional inference

- *Theorem 1. Given monotonic* $\neg$, $\oplus$, *and* f, *the upward pass and downward pass converges within* ϵ *in* ***finite time***.
- *Q: Does this hold in FOL?*



Ref: https://en.wikipedia.org/wiki/Cauchy_sequence

Reason under the hood: Cauchy sequence

What

What's the main difference? - Bidirectional inference

- *Q: Does this hold in FOL? No.*
  But LNN works in FOL, how does it do? (Skip)

What

What's the main difference? - Bidirectional inference

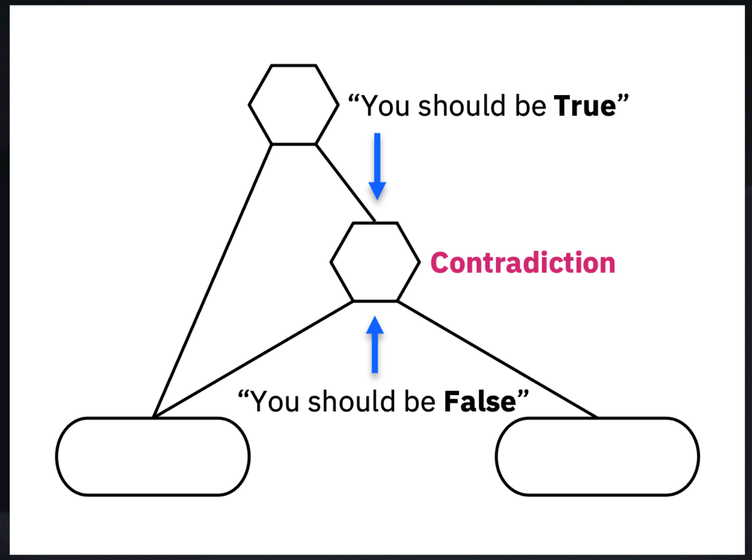
- *Theorem 2 (Modified). The* $\boldsymbol{L}$ *and* $\boldsymbol{U}$ *on any formulae are concrete/sound bounds on* ***possibility*** *based on all the previous acquired knowledge.*

- A sound and probabilistic reasoner.
  Shows the adaptive ability to **open-word assumptions** with incomplete knowledge compared to classical Markov Logic Networks. ("unknown"!)

Learning

A bit of the power on leaning

- Allows for contradiction (Logic inconsistency) $L_x > U_x, T_x > L_x, T_x < U_x$
- In propositional logic, what is the representation of the logic inconsistency?



Ref: https://github.com/IBM/LNN/tree/tutorials/tutorials

Learning & Showcase

Wait, weights?! Live coding!

- 👀 See how it works in the Cat-Dog case.
- ✍🏽 See how it works with example of conjunction.

Resources

IBM's archive

- An overview of all related projects. (Some of them are not properly archived)
- NSTK

Thank you